West Antarctic balance calculations: Impact of flux-routing algorithm, smoothing algorithm and topography

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Abstract

Balance flux and velocity calculations are important in understanding the large-scale dynamics of ice masses and their state of balance. However the grid-based (finite difference) nature of the current algorithms means that there is a variety of ways in which the balance flux and velocity can be calculated. The flux-routing algorithm, grid orientation and grid size all have an impact on the balance calculations. Previous work has relied on the assumption that the ice flow direction is orthogonal to the surface contours, with the surface having been smoothed to incorporate a representation of the longitudinal stresses within the ice. This assumption is a simplification of the gravitational driving stress equation, which relates the driving stress to the gradient of the surface slope and the ice thickness. Further, the common representation of longitudinal stresses using a fixed size smoothing filter is a simplification of the theoretical treatment of longitudinal stresses by Kamb, B., Echelmeyer, K.A. [1986. Stress–gradient coupling in glacier flow: 1. longitudinal averaging of the influence of ice thickness and surface slope. Journal of Glaciology 32 (111), 267–298]. This study investigates the sensitivity of the balance calculations to these issues, using the West Antarctic Ice Sheet as a case study. Significant differences in both ice stream margins and values of ice flux within them result from the algorithm choice, grid orientation and grid size. The incorporation of a theoretically appropriate smoothing approach improves the coherence of the pattern of ice flow. The incorporation of gravitational driving stress has a small effect in comparison to practical issues such as the grid orientation and size. As higher resolution datasets (less than 5 km) become available for many ice masses, there is a temptation to calculate the balance flux using the same algorithms on the finer grids. The results shown in this study suggest that the type of algorithm most commonly used is not suitable for these finer grids. The impact of the practical issues described here encourages caution in the use of grid-based balance distributions and values, especially when considering the state of balance of individual ice streams.

Keywords: Balance flux; Balance velocity; West Antarctic ice sheet; Routing algorithm; Smoothing filter

1. Introduction

Grid-based calculations of balance flux and velocity are widely used to understand the dynamics and the state of balance of ice sheets. The balance flux at a given point is the depth-integrated ice flux (m² yr⁻¹) needed to balance the ice input
(accumulation) upstream of the point. If the ice thickness is known, this flux can be converted to a depth-averaged velocity.

Before extensive Interferometric Synthetic Aperture Radar (InSAR) measurements of velocity were available, balance velocities provided a means to understand ice flow over an entire ice sheet (e.g. Budd and Warner, 1996 (Antarctica); Joughin et al., 1997 (Greenland); and Bamber et al., 2000 (Antarctica)). They have been used in planning locations for fieldwork, as an initial condition or validation for modelling (e.g. Budd and Huysters, 1996) and as a source of tie points for interferometric phase unwrapping (e.g. Shepherd et al., 1997). The aim of this study is to investigate whether the numerical schemes have a significant effect on the flux calculations. Flux routing algorithms, designed for use in hydrology, will be assessed for robustness in ice balance calculations. The impact of other choices made in the flux calculations on the resulting distribution will also be investigated. These choices include the approach chosen for smoothing the topography and the topographic dataset chosen. The algorithms will be applied to the West Antarctic Ice Sheet (WAIS), with particular reference to the values within the ice streams (Fig. 1).

2. Balance concept

The balance assumption is that the mass of ice flowing out ($\Psi_{\text{out}}$, m$^3$ yr$^{-1}$) of any area $S$ (m$^2$) within the horizontal domain $(x,y)$ exactly equals the sum of the inflow ($\Psi_{\text{in}}$, m$^3$ yr$^{-1}$) and the ice accumulated over the area (Budd and Warner, 1996; Fricker et al., 2000):

$$\Psi_{\text{out}} = \Psi_{\text{in}} + \int_{S} a(x,y) \, dx \, dy,$$

where $a$ is the accumulation (m yr$^{-1}$).

It then follows that the integrated accumulation over area $S$ is equal to the net volume of ice that flows across $C$, the boundary of $S$; therefore (1) can be written as

$$\int_{C} H \nabla \cdot \vec{n} \, dk = \int_{S} a(x,y) \, dx \, dy,$$

where $H$ is the ice thickness (m), $\nabla$ is the depth averaged velocity (m yr$^{-1}$), $\vec{n}$ is the outward directed unit vector normal to the boundary and $k$ is the distance along $C$ (m).

The quantity

$$\psi_{\text{net}} = \int_{C} H \nabla \cdot \vec{n} \, dk$$

is a scalar flux representing the volume (m$^3$ yr$^{-1}$) flux through $C$. The term “balance flux distribution” ($\Phi_B$) is a vector flux field given for a unit width (m$^2$ yr$^{-1}$) in the direction of flow:

$$\Phi_B = H \nabla.$$

This distinction between scalar flux and vector flux is important and causes a number of difficulties when dealing with the nature of grid boundaries.

3. Steps in grid-based balance calculations

In order to obtain a grid-based flux distribution, Eqs. (1–4) need to be solved discretely. The algorithms used in this paper use a finite difference scheme on a rectangular grid. The steps in this process are shown in Table 1 and the following discussion elaborates on the options available at each step.

Step 1. Source of flow direction and
Step 2. Smoothing approach.
In order to determine the route accumulation takes to an outlet, the flow direction needs to be calculated. Provided that the bed and surface slopes are small, and the horizontal scale is greater than 10–20 times the ice thickness, the following equation holds for the driving stress of ice (Paterson, 1994):

\[ \tau_x = \rho g H \frac{\partial s}{\partial x} \]  

solved on a finite difference grid, with an analogous expression in the y direction, where \( \tau_x \) is the
gravitational driving stress (GDS), \( \rho \) is ice density, \( g \) is acceleration due to gravity, and \( s \) is the surface elevation.

Eq. (5) implies that the ice flow will be parallel to the steepest gradient of the surface topography. This assumption is key to calculations of balance flux which use surface topography to derive flow direction. The surface elevation \( s \) is first smoothed over a number of ice thicknesses, in an attempt to incorporate the effects of longitudinal stresses which smooth out localised variation in the ice surface. However, Kamb and Echelmeyer (1986) state that both variations in the surface slope \( \partial s / \partial x \) and the ice thickness \( H \) determine the effect of longitudinal averaging on local flow. Therefore, the full gravitational driving stress (including \( \partial s / \partial x \) and \( H \)) must be smoothed, rather than only the surface elevation \( s \). The full driving stress can be incorporated by calculating \( x \) and \( y \) components to derive flow direction. The \( x \) and \( y \) components can then be smoothed independently (Step 2).

**Step 3. Smoothing technique.**

Kamb and Echelmeyer (1986) show that the influence of slope and thickness perturbations drops off exponentially with distance. The exponential decay has a length scale \( l \), which they term the ‘longitudinal coupling length’

\[
W_l = e^{-\frac{1}{2}(x-x')^2+(y-y')^2} 
\]

where \( W_l \) is the weighting function, \( x \) and \( y \) are the co-ordinates of the point being smoothed and \( x' \) and \( y' \) are the co-ordinates of the weighted points. The weighting functions in Fig. 2 are scaled to have unit area over the interval \(-2l\) to \(2l\) (leading to an “averaging length” of \(4l\)), as a practical approxima-

tion of the exponential weighting. This exponential decay means that the local slope will have the largest influence on ice flow, but will be “muted” by the spatial averaging. Previous balance calculations have used box-car averaging (either fixed size or variable size based on the ice thickness, Fig. 2); however, this gives equal weighting to cells irrespective of their distance from the point in question and causes an abrupt cut-off point where the weighting falls to zero. Due to varying ice thickness, a fixed-size filter also leads to differing coupling lengths for different areas of the ice sheet. Kamb and Echelmeyer (1986) indicate that a linear (or chapeau) weighting function is acceptable as a practical approximation to exponential decay (Fig. 2).

**Step 4. Flux routing algorithm.**

Flow routing algorithms are commonly used in hydrological applications, primarily for calculating upstream accumulation areas. A number of different algorithms have been developed and tested for this purpose (see Tarboton, 1997). There are two distinctions between the routing algorithms. First, the routing algorithm can allow flow into multiple neighbouring cells or just a single neighbour. In multiple-flow direction algorithms, one unit (cell) of ice can flow into any number of neighbours that are down-slope. With single flow direction algorithms, the unit of ice remains coherent and can only flow into one cell. Multiple flow direction algorithms introduce significant dispersion which is inconsistent with upslope area (only one cell should be able to “claim” the upslope cells) (Tarboton, 1997). Quinn et al. (1991) acknowledge that multiple flow direction algorithms result in different distributions compared with single flow direction algorithms, and
that these differences will become larger as the size of the cell, and the likelihood of significant curvature of contour lines within a cell, increases: “In essence the multiple flow direction algorithm is a simple and approximate form of sub-grid scale flow pathway interpolation” (Quinn et al. 1991). Single flow direction algorithms are not presented in this investigation, as they do not produce a realistic distribution of ice flux.

The second distinction is whether the routing algorithm allows flow into any of four or eight neighbours. A standard finite difference scheme considers a cell to have only four neighbours (adjacent neighbours), and does not include the diagonal neighbours. Budd and Warner (1996) found that using all eight neighbours reflects curved topography better when using coarser grids.

All the algorithms use a centred difference method on a square grid; therefore each cell can be considered as area $S$ above. Each cell edge can have only inflow or outflow across it, the balance equation (scalar flux) for each cell is therefore

$$\psi_{ij}^\text{out} = \psi_{ij}^\text{in} + a_{ij}r^2,$$

where $r$ is the cell length (assuming a square grid).

The outflow from each grid cell is apportioned between its neighbours, depending on the nature of the algorithm (see Fig. 3 for a summary). In all algorithms, the total scalar inflow of ice into each grid cell is the sum of the out-flowing contributions from neighbouring up-slope cells.

The flow routing algorithm employed by Budd and Warner (1996, henceforth known as the “Warner” routing algorithm) and that employed by Quinn et al. (1991, henceforth known as the “Quinn” routing algorithm) are similar in that they both apportion the flux by dividing the total outflow from the cell in proportion to the elevation differences with each neighbour. The Quinn routing

Fig. 2. Weighting functions for longitudinal averaging (after Kamb and Echelmeyer, 1986).

Fig. 3. Schematic diagram to show cells which can receive flux from cell$_{ij}$ (light grey) and those that will receive flux given general flow direction indicated by grey arrow (dark grey), under different algorithms: (a) Warner, (b) Quinn and (c) Tarboton.
algorithm considers all eight neighbours, whereas the Warner routing algorithm only considers the four adjacent neighbours. Therefore the contributing flow from cell_{ij} to cell_{i,j+1} for the Warner and Quinn routing algorithms is
\[
\psi_{i,j+1}^{in} = \frac{h_{i,j} - h_{i,j+1}}{\sum \Delta h} \psi_{i,j}^{out},
\]
where \( h_{i,j} \) is the surface elevation in cell_{i,j} and \( \sum \Delta h \) is the sum of the elevation differences for all downslope cells.

The routing algorithm described in Tarboton (1997, henceforth known as the “Tarboton” routing algorithm) fits triangular facets to the eight neighbours in order to derive a slope to each neighbour. The flow direction is defined by the steepest downslope vector from all eight facets, and flow is assigned to two cells (or one cell, where the direction follows the cardinal direction) based on the proportions of the angles. A modified version of this algorithm (“modified Tarboton” routing algorithm) obtains the flow direction from the local slope of the topography using the four cardinal neighbours. This routing algorithm is included in the discussion because it forms the basis of incorporating the smoothed gravitational driving stress (GDS).

A further modified version of the Tarboton routing algorithm (“GDS–Tarboton” routing algorithm) obtains the flow direction from the smoothed gravitational driving stress calculated in \( x \) and \( y \) (see Eq. 4). The Warner routing algorithm can also be modified to accept the smoothed gravitational driving stress in \( x \) and \( y \) (“GDS–Warner” routing algorithm). Here the flux can only be routed into a maximum of two neighbours.

Step 5. Flux solving algorithm.

In order to calculate the balance flux contribution from all cells to each individual cell, some form of flux routing solver is required. There are three possible options: (a) matrix representation, (b) elevation sort and (c) recursion. A matrix holding the contribution of each cell to every other cell can be solved to yield a matrix of flux though each cell. This is inefficient in terms of computational memory (even if sparse matrix methods are used) and expense. By sorting the cells by elevation, and beginning with the highest cell, the cell outflow rates can be found sequentially. For each cell, the total flow into the grid cell is already known as the outflow for its upslope neighbours will have already been calculated. However, this solving algorithm relies on the assumption that flow can only occur down-slope, which does not always hold when using the smoothed driving stress as the source of flow direction. In the recursive technique, each cell calls the procedure for each contributing neighbour, and so on recursively to the ice divide (which has no contributing neighbours). The flux is then first calculated at the divide and then for each cell it contributes to, back down towards the outlet.

This flux-solving technique can be used for all the flow routing algorithms and is not computationally demanding.

Two further steps are needed before vector balance flux \( \text{m}^3 \text{yr}^{-1} \) can be calculated.

3.1. Hollow filling

A hollow-filling algorithm must be applied to the topography to ensure there are no sinks for the flux. Sinks are likely to be spurious features in the data, due to processing and hence can be filled. Hollow filling is carried out iteratively by assigning the “sink” cell the value of its lowest neighbour (considering either 4 or 8 neighbours depending on the algorithm).

3.2. Scalar to vector flux conversion

The vector balance flux (quantity \( H V \), Eq. 4) at a point has units of \( \text{m}^3 \text{yr}^{-1} \). The flux routing algorithms calculate the scalar flux \( \text{m}^3 \text{yr}^{-1} \) because it makes the calculations simpler (the nature of the flux cross-section is then not important, it is a function of the grid size). In order to convert the scalar flux to a vector flux field, the length of the flux cross section now needs to be considered. If an overall flow direction for the cell is obtained from the topography, the flow direction can be used to convert the scalar flux into a vector flux (see Budd and Warner, 1996).

The direction relative to the grid orientation \( \theta \) is calculated by fitting a plane to the elevations of the four neighbouring cells. Taking the outflow from a cell as a good approximation to the flow through it, the flux magnitude \( \Phi_B \) is obtained by
\[
\Phi_B = \frac{\psi_{i,j}^{out}}{r(\cos \theta) + \sin \theta)}.
\]

The same conversion method is used for all algorithms to ensure it has no impact on the flux routing algorithm comparison.
4. Datasets

Two or three input datasets are needed for the balance calculations, depending on the options chosen and the balance quantity required (flux or velocity). The input data required for balance flux calculations are surface topography and accumulation. Ice thickness data are also required for balance velocity calculations. The thickness dataset is also needed for smoothing the topography (using the variable weighting filter) or for calculating the driving stress for both flux and velocity calculations.

There are a number of Digital Elevation Models (DEMs) available for Antarctica. The first satellite-derived DEM was created by Bamber and Bindschadler (1997, termed JLB97) using ERS-1 (European Remote Sensing) radar altimetry data at a 5 km resolution. A second DEM was later created using these data, for correction of the Radarsat Antarctic Mapping Project (RAMP) image mosaic (Liu et al., 1999). Liu et al. (1999) used a different processing algorithm and supplemented it with cartographic and radio echo sounding-(RES) based sources. Data from the geoscience laser altimeter system (GLAS) instrument on board the IceSat satellite platform has allowed a validation of these data sources and shown that the RAMP DEM has significant errors where cartographic data has been used (Bamber and Gomez-Dans, 2005). The GLAS data have also been used to create a further DEM (Smith, 2004, pers. comm. and Gomez-Dans, 2005, pers. comm.). The quality of the topographic dataset is not of primary importance for this paper, and so the RAMP dataset was chosen because it has been published and is available at a high resolution (up to 200 m). The impact of the topography dataset is discussed in Section 9.

The snow accumulation dataset used in this work was interpolated from point measurements of accumulation, using passive microwave satellite data to control the interpolation (Vaughan et al., 1999). The dataset has a resolution of 10 km.

The ice thickness dataset used is from the BEDMAP project (Lythe et al., 2001). This dataset combines different types of data, primarily using airborne and ground-based RES, but also using seismic and gravimetric measurements. Large parts of the WAIS have poor coverage (particularly the Amundsen Sea area) so the data are likely to be fairly inaccurate around these areas. The dataset has a resolution of 5 km.

An initial resolution of 5 km was chosen for the modelling due to the nature of the datasets, and for ease of computation. To produce the necessary datasets, bilinear interpolation was used to resample the 1 km RAMP DEM dataset to 5 km, and the accumulation dataset from 10 to 5 km.

5. Structure of the results

The effects of employing different options for the various stages in the balance calculations will now be investigated. First, the impact of the flux routing algorithm on the flux distribution will be investigated for both simple test cases and for West Antarctica. The effect of the grid orientation and resolution on the flux distribution arises from this discussion. Second, the impact of different topography smoothing techniques on the flux distribution will be investigated using the Warner routing algorithm. Third, the impact on the flux distribution of using the gravitational driving stress to derive the flow direction will be investigated by using modified versions of the Warner and Tarboton routing algorithms. Finally, the uncertainty introduced to the flux distribution by errors in the topography dataset will be investigated. The impact of the flux solving algorithm on the flux distribution is not discussed here as its impact is not significant compared to the other factors discussed. The flux values quoted are in \( \text{m}^3 \text{yr}^{-1} \) water equivalent (w.e.).

6. Impact of routing algorithm

In order to evaluate the effect of the algorithm on the flux values, it is useful to consider a flux profile across an ice stream. There are two important properties of such a profile that need to be considered; the shape of the profile, and the total flux through the profile. If the balance velocity is compared with a measured velocity profile, then it is important that the shape of the profile is consistent for each algorithm. If the total flux across the profile is required, it is important that the algorithm provides the correct flux total across the profile. These criteria provide a quantitative method to test the algorithms and give an indication of the success of the algorithm. The three algorithms will therefore be discussed with these criteria in mind.

Before considering the flux results in a WAIS context, it is important to understand how each algorithm works and hence what the flux values in each cell mean. The three algorithms were tested in
a series of experiments using a planar surface, with the flow-oriented parallel to the grid orientation through to 45° to the grid orientation. Influence maps (flux pattern resulting from one unit of accumulation added at the top of each slope) were constructed for each routing algorithm (Fig. 4). From these test cases four issues relating to the operation of the algorithms become apparent. First, the Warner routing algorithm leads to severe grid orientation effects (Fig. 4(a)). When the flow is parallel to the grid orientation, it is confined to a channel of one cell width, however when flow is oriented at 45° to the grid orientation, the flow spreads over a number of cells. This is less of a problem for the Tarboton routing algorithm (Fig. 4(c)), and not a significant problem with the Quinn routing algorithm (Fig. 4(b)). As a result, when the Warner routing algorithm is used, grid orientation will be a very important factor in the shape of a resulting flux profile, (though it should be noted that grid orientation does not affect the overall flux total across the profile). Second, different levels of dispersion are inherent in each algorithm. The Tarboton routing algorithm shows the least dispersion of the three, the Quinn routing algorithm shows the most dispersion regardless of orientation. The Warner routing algorithm shows dispersion when the flow direction moves away from parallel to the grid orientation. Third, when flow is at 20° to the grid, the general angle of flow varies between the three algorithms. A profile drawn across the base of each plane would show that the peak flux would occur at a different point along the profile. Fourth, when the flow direction is at 45° to the grid orientation, the total flux across a profile drawn perpendicular to the flow direction is higher for the Warner routing algorithm than the Quinn routing algorithm. Depending on where the profile is drawn this may also be the case for the Tarboton routing algorithm.

The best (though idealised) way of understanding how the routing algorithm impacts on the flux total is to consider flux across a flat plane, aligned diagonal to the grid orientation (Fig. 5). One unit of accumulation (here in scalar flux (m³ yr⁻¹) to make the concept simpler, hence considering cells rather than points) is added in the top left corner cell (cell 1). In the case of the Warner routing algorithm (Fig. 5(a)) half the flux goes into each of the down-slope neighbours (2 and 4). With the Quinn routing algorithm however, part of the flow goes directly into the corner neighbour (5, Fig. 5(b)). If a cross section was drawn at Pl, the total flux for the Warner routing algorithm would be 1 and the flux

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Fig. 4. Influence maps for 1 unit of accumulation (m³ yr⁻¹; scale 1 (black) to 0 (white)) added at top of a planar surface oriented at various angles to grid. (x, y and z arbitrary units—number of cells).
would be conserved, whereas for the Quinn routing algorithm, the total flux at $P_1$ is only 0.666. This is because the direct flow (0.333) into the corner cell is unaccounted for. Similarly at $P_2$, the flux for the Warner routing algorithm is 1 compared to 0.778 for the Quinn routing algorithm (note, there are two lots of 0.111 unaccounted for). This problem is even more significant for the Tarboton routing algorithm as more flow is likely to go into the diagonal neighbours, in our extreme case in Fig. 5, a profile could be drawn that yields a zero flux value ($P_1$). It is important therefore to consider the individual flux values differently for each algorithm. When considering flux at a point ($m^3 yr^{-1}/C_0$) the same concept applies, but the flux across the profile must be calculated appropriately.

6.1. WAIS flux pattern

The three algorithms were applied to the RAMP DEM (Fig. 6), smoothed using a chapeau smoothing filter (which has a longitudinal coupling length of 5 times the ice thickness). The flux results demonstrate the effect that the flow algorithm has on balance flux calculations. The overall location of the ice streams does not vary with different algorithms, as the main control on the ice streams is the surface topography (which is unchanging). However, the form of some of the flux features does change with algorithm, especially those which follow the grid orientation. In terms of the pattern of the flux distribution, the Quinn routing algorithm gives a much smoother distribution, especially for features aligned parallel to the grid orientation (e.g. Institute Ice Stream tributaries). The Tarboton routing algorithm generally gives narrower ice streams.

6.2. WAIS flux values

6.2.1. Effect of grid orientation

Due to the flow configuration of Pine Island Glacier and the majority of its tributaries being aligned almost parallel to the grid on the original DEM, this ice stream is used as a case study to illustrate the points identified in the test cases when considering a profile. The flux values were corrected for their direction relative to the profile. The Warner and Tarboton routing algorithms result in a similar profile on the un-rotated dataset, whereas the Quinn routing algorithm yields a wider stream with a much lower peak value (Fig. 7(a)). The difference in the profile shape can be understood by considering the influence maps for the test cases where flow is oriented parallel to the grid (Fig. 4, 0°). The Quinn routing algorithm leads to more disperse flow, hence has a wider profile with a lower peak. The flux totals across each profile are similar (Table 2, Pine Island Glacier, 0° orientation), with slightly higher values for the Warner routing algorithm.

The datasets were then rotated by 20° and 45°, and the flux was calculated again using each algorithm (Fig. 7(b and c)). Different interpolation methods were tested for the re-sampling but this had little impact on the resulting flux distribution. At 20° rotation, the resulting flux profiles for each routing algorithm are more similar than using the original dataset, although the peak values occur at different points along the profile. At 45° rotation the profiles resulting from the Warner and Quinn routing algorithms are very similar, although the values resulting from the Quinn routing algorithm are consistently lower. The flux totals are also different, with the flux totals resulting from the
Quinn and Tarboton routing algorithms being much lower than the Warner routing algorithm (Table 2, Pine Island Glacier, 45° orientation). At 45° rotation, the flux total resulting from the Quinn routing algorithm is about 77% of that resulting from the Warner routing algorithm. This corresponds to 79% of the expected flux total for example profile $P_2$ in Fig. 5. The grid orientation

### Table 2

Flux totals ($\times 10^9$ m$^3$ yr$^{-1}$) and peak values ($\times 10^6$ m$^2$ yr$^{-1}$) for profiles across various ice streams (shown on flux distribution figures)

<table>
<thead>
<tr>
<th>Ice stream</th>
<th>Orientation</th>
<th>Warner</th>
<th>Quinn</th>
<th>Tarboton</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Flux total</td>
<td>Peak value</td>
<td>Flux total</td>
</tr>
<tr>
<td>Pine Island Glacier</td>
<td>0</td>
<td>52.4</td>
<td>7.45</td>
<td>48.9</td>
</tr>
<tr>
<td></td>
<td>20</td>
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<td></td>
<td>45</td>
<td>52.1</td>
<td>2.58</td>
<td>40.0</td>
</tr>
<tr>
<td>Thwaites</td>
<td>0</td>
<td>58.3</td>
<td>1.42</td>
<td>52.5</td>
</tr>
<tr>
<td></td>
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<td>58.7</td>
<td>1.01</td>
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<td>2.66</td>
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<td></td>
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<td>0.44</td>
<td>15.0</td>
</tr>
</tbody>
</table>
effects of the Warner and Tarboton routing algorithms are made clear by the profiles resulting from each orientation, whereas the grid orientation has little effect on the resulting profiles from the Quinn routing algorithm.

The problems outlined above for Pine Island Glacier are transferable to the other ice streams (Table 2). Thwaites Glacier shows a similar response to grid orientation to Pine Island Glacier, with the peak flux values more similar at 20° and 45° of rotation. For Evans Ice Stream, the greatest difference in peak flux values across the algorithms occurs at 20° rotation. The downstream end of Evans Ice Stream is aligned roughly parallel to the grid; however it has tributaries at various flow angles, so it would be expected that grid orientation would have less impact on this ice stream. The Quinn routing algorithm is the most consistent of the routing algorithms, in terms of both the flux total across the profile and the peak value of the profile, across all grid orientations. The flux totals from the Quinn routing algorithm are consistently lower than the totals from the Warner routing algorithm.

Overall, the flux totals (across the profile) resulting from the Warner and Quinn routing algorithms are independent of the grid orientation (although the flux totals resulting from the Quinn routing algorithm are consistently lower than using the Warner routing algorithm). However, the Quinn routing algorithm does not show consistent flux totals across all grid orientations for Pine Island Glacier. This is a fairly special case because of the orientation of its tributaries, therefore when the stream and tributaries are oriented at 45° to the grid, the routing algorithm suffers significantly from the problems demonstrated in Fig. 5. For ice streams that have tributaries at various angles to the grid orientation (e.g. Evans Ice Stream) this is not as significant. The Tarboton routing algorithm shows little consistency in the flux totals.

6.2.2. Effect of grid size

The measured velocity (InSAR, Shepherd et al., 2001) for Pine Island Glacier along the transect (shown on flux distribution figures in white) is about 1500 m yr\(^{-1}\) (the stream covers most of the transect). Assuming an ice thickness of 1700 m (from BEDMAP, though the accuracy here is questionable, Thomas et al., 2004) along this transect we would expect the peak flux to be approximately \(2.32 \times 10^6\) m\(^2\) yr\(^{-1}\) (w.e.). On the un-rotated grid, the peak flux for the Warner routing algorithm at 5 km is \(7.44 \times 10^6\) m\(^2\) yr\(^{-1}\), whereas at 20 km resolution it is \(2.24 \times 10^6\) m\(^2\) yr\(^{-1}\). At a 1 km resolution the flux increases to slightly over \(30 \times 10^6\) m\(^2\) yr\(^{-1}\) (Fig. 8(a)), and the ice stream is effectively reduced to 4 km wide (compared to about 40 km from InSAR). This suggests that a grid resolution of 20 km is an appropriate resolution to provide a reasonable estimate of a balance velocity profile for Pine Island Glacier at this orientation. Pine Island Glacier is an extreme case due to its configuration and width, but the effect of changing grid size is replicated over all the other ice streams. The impact is less spectacular in the case of Thwaites Glacier (Fig. 8(b)), a less well constrained ice stream. The expected peak flux for Thwaites Glacier from the InSAR velocity and thickness data is \(2.05 \times 10^6\) m\(^2\) yr\(^{-1}\). Even with a resolution of 1 km (peak flux \(\sim 1.93 \times 10^6\) m\(^2\) yr\(^{-1}\)), the measured peak flux is not reached by the balance calculations. The fact that the measured and modelled velocities are not similar at a 20 km resolution for Thwaites Glacier shows it is important to avoid trying to calibrate the balance flux and velocity, using the measured velocity, in order to find the most appropriate resolution. Testut et al. (2003) also found that the peak velocity varies with grid resolution. Their comparison of measured velocity with balance velocity suggests that a 5 km resolution produces velocity peaks that are much higher than
measured. The relative success of a 20 km resolution dataset is also demonstrated by Wu and Jezek (2004).

The narrowing of the ice streams at a higher resolution suggests that the design of the algorithms is not appropriate for high-resolution balance calculations. In hydrological applications, the features (channels) are generally sub-grid features, whereas a 5 km grid resolution in ice sheet applications (e.g. Bamber et al., 2000) means a typical ice stream is composed of around five to ten grid cells. This may lead to the channelling of flow into the centre of the ice stream and narrowing of its profile. When the resolution is increased to 1 km (tempting given the recent release of high-resolution topography datasets) then the effect is increased still further.

7. Impact of smoothing filter

The impact of the type of smoothing filter on the balance calculations was investigated by applying the Warner routing algorithm to the DEM (5 km RAMP DEM) smoothed with different filters. Kamb and Echelmeyer (1986) suggest that the longitudinal coupling length for ice sheets should be between four to ten times the ice thickness. The average ice thicknesses in the WAIS is approximately 1300 m. Using this value of ice thickness, a coupling length of four times the ice thickness corresponds to an averaging length \( l \) of approximately 21 km. Ten times the ice thickness corresponds to an averaging length of 52 km. Fricker et al. (2000) use a fixed-size (35 km) filter for the study of the Lambert Glacier basin (East Antarctica). Assuming that ice thickness varies between 1000 and 4000 m in this area, this corresponds to \( 2 < l < 8 \). Testut et al. (2003) tested various sized box-car filters, choosing a 100 km filter for their comparison. This corresponds to \( 6 < l < 25 \), much higher than that suggested by Kamb and Echelmeyer (1986). It is suggested that such a large smoothing filter was needed to counter the effect of a 5 km grid resolution.

A fixed-size box-car averaging filter was tested, with a size of 25 km \( (l\sim4.8) \), 35 km \( (l\sim6.7) \) and 45 km \( (l\sim8.7) \) (Fig. 9). A reasonably smooth flux distribution emerges with a filter size of 35 km. With a filter size less than 35 km, the pattern of tributaries is not always sufficiently smooth (e.g. Thwaites Glacier tributaries).

Due to varying ice thickness over the WAIS, an improvement on the fixed-size filter is a variable-size box-car filter. Using \( l = 5 \) (averaging length = 20 times the ice thickness, Fig. 10(a)), the flux pattern of some of the features (e.g. Pine Island Glacier tributaries) differs from the flux pattern resulting from the fixed-size box-car filter. The box-car nature of the filter gives a slightly blocky appearance and, as discussed in Section 3 (step 3), is not theoretically acceptable. The variable-weighting, variable-size filter (chapeau) gives much the same distribution as the variable-size box car filter, however it gives a much smoother looking distribution (Fig. 10(b)). Using \( l = 10 \) causes many of the tributaries to become “smeared” and lose their form, suggesting this coupling length is too large.

![Fig. 9. WAIS flux distribution (m² yr⁻¹ w.e.) calculated from topography smoothed with varying sized filters: (a) 25 km, (b) 35 km and (c) 45 km.](image-url)
8. Source of flow directions

In this section, the flow direction is derived from the smoothed gravitational driving stress in $x$ and $y$. There are two potential methods of incorporating the driving stress into the routing algorithm; using the modified Tarboton routing algorithm or a modified Warner routing algorithm (see Step 4, Section 3). The two algorithms were applied to driving stress components smoothed using the chapeau filter ($l=5$). Both algorithms give a very smooth and coherent flux distribution (Fig. 11), but have only a small effect on the location of, and flux values in the ice stream (e.g. Fig. 12). The GDS routing algorithms suffer from the same problems with grid orientation and grid size as the respective slope-based algorithms outlined in Section 6.

The flux distributions in Fig. 9 result from the traditional Budd and Warner (1996) approach, using ice elevation smoothed with a fixed size, box-car filter. Fig. 11(b) uses a similar flux routing algorithm, but derives the flow direction from the gravitational driving stress smoothed using a variable-size, variable-weighting filter. The flux distributions shown in these figures are very different. Figure 11(b) shows a much smoother, more coherent pattern of flux.

Fig. 10. WAIS flux distribution (m$^2$ yr$^{-1}$ w.e.) calculated from topography smoothed with different filters: (a) box car, $l=5$, (b) chapeau, $l=5$ and (c) chapeau, $l=10$.

Fig. 11. WAIS flux distribution (m$^2$ yr$^{-1}$ w.e.) calculated using gravitational driving stress: (a) GDS–Tarboton and (b) GDS–Warner.
9. Topography

Due to geographic limits on satellites and problems in areas of high slope, topographic datasets of the WAIS contain some error which will introduce further uncertainty into the balance calculations. The impact of different topography datasets was investigated using the GDS–Warner routing algorithm, applied to the three available surface topography datasets (Fig. 13). The most notable difference of the three resulting flux distributions is the Rutford Ice Stream. In the RAMP DEM the flux flowing around the Ellsworth mountains is diverted into Carlson Inlet, whereas in both the JLB97 and GLAS DEMs the flux is correctly routed into Rutford Ice Stream. Comparison of the RAMP DEM with the GLAS DEM indicates that there is a discrepancy in height of up to 200 m at the divergence of flow between Rutford Ice Stream and Carlson Inlet, causing diversion of the flow into Carlson Inlet. This difference demonstrates the importance of the quality of the input dataset. A poor quality surface topography dataset will lead to erroneous flux predictions, whatever the nature of the algorithm.

The difference in location of Rutford Ice Stream also suggests that a slowing and consequent thickening of Rutford Ice Stream could cause flux to be diverted into Carlson Inlet and ultimately lead to the stagnation of Rutford Ice Stream. This scenario is analogous to the capture of Ice Stream C (Kamb) flux by Ice Stream B (Whillans) (e.g. Anandakrishnan and Alley, 1997) and further indicates the potential dynamic nature of the WAIS.

10. Conclusions

Balance flux (or velocity) of an ice-sheet is considered to be an inherent property of an ice
sheet. Balance flux is essentially equivalent to upslope catchment area, and hence it is reasonable to calculate the balance flux using routing algorithms designed for calculating upslope catchment area. The range of results shown by Tarboton (1997) for upslope area and by this study for balance flux suggests that no grid-based algorithm can give a “correct” calculation of this quantity. This study has attempted to investigate the reasons for the different results and find the most appropriate algorithm for calculating ice balance flux and velocity. The results have shown that the flux distribution resulting from routing algorithms that consider a cell to have eight neighbours needs to be interpreted in a different way to the flux distribution resulting from routing algorithms which consider only four neighbours. The flux distribution profile resulting from the Quinn routing algorithm can underestimate total flux (i.e. lose flux) by anything up to 25% and the Tarboton routing algorithm by up to 50%. This loss of flux is most pronounced when flow is aligned diagonally to the grid orientation. The flux distribution resulting from the algorithms is also affected by the orientation and resolution of the grid. The choice of the most suitable grid resolution is subjective, and appears to be at a lower resolution than is desirable for most purposes. Further subjectivity is introduced by the choice of the longitudinal coupling length. The type of smoothing filter is constrained by theory (Kamb and Echelmeyer, 1986), but the choice of coupling length is constrained only by inspection of the resulting flux distribution.

This study has highlighted the potential pitfalls in the calculation of grid-based balance flux and velocity, although the distributions generated from this type of algorithm can still be of use. Firstly, the overall flow pattern is, on the whole, unaffected by the choice of routing algorithm, etc. as this is reliant only on the surface topography. The GDS–Warner algorithm is proposed for the purpose of understanding the pattern of ice flow. Secondly, the flux total across a profile resulting from the Warner routing algorithm (both original and GDS) is consistent across different orientations and resolutions. It is when the shape of the flux profile is considered that inconsistencies arise with all algorithms. A large amount of uncertainty is then introduced by the choice of algorithm, smoothing technique, grid resolution and orientation and, in the case of West Antarctica, by errors in the topography and ice thickness datasets. This uncertainty could lead to erroneous conclusions in the use of grid-based balance calculations for the purpose of comparison with measured velocity.

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Appendix A. Supplementary materials

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.cageo.2006.04.005.

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